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INCLUDING PARAMETER UNCERTAINTY IN FORWARD PROJECTIONS OF COMPUTATIONALLY INTENSIVE STATISTICAL POPULATION DYNAMIC MODELS

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Abstract
The increased computational demands of modern statistical stock assessment models has made the standard methods to provide uncertainty estimates for forward projections impractical in many applications. However, forward projections and their associated estimates of uncertainty are an important and popular piece of management advice. We describe a less computationally intense method to estimate uncertainty in forward projections that includes both parameter uncertainty and future demographic stochastic uncertainty. This method is based on extending the estimation model to include the future projection period and using normal approximation based on the Hessian matrix to estimate confidence intervals. The method is tested using simulation analysis and compared to Bayesian integration and to projections based on the point estimates of the parameters. We identify bias caused by the lognormal penalty on annual recruitment deviations, the lognormal bias correction factor for recruitment and effort deviations, and the stock recruitment relationship. We suggest modifications to eliminate the first two biases. The method is applied to yellowfin tuna in the eastern Pacific Ocean.

Introduction

Modern quantitative methods used to assess the status of commercial fisheries have become very complex and computationally intensive (Quinn 2003). These methods are usually statistical, nonlinear, and can have hundreds or thousands of parameters (e.g., Fournier et al. 1998; Hampton and Fournier 2001; Butterworth et al. 2003; Maunder and Watters 2003). Sophisticated iterative nonlinear function optimizers are needed to estimate the model parameters and provide management advice. These analyses often take days to estimate the parameters, which limits the amount and type of analyses that can be performed. For example, only a limited number of sensitivity analyses can be performed to investigate the model's robustness to structural assumptions.

A current trend in fisheries stock assessment is the estimation and presentation of uncertainty (Francis and Shotton 1997; Punt and Hilborn 1997). Consideration of uncertainty is essential when

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making management decisions and is particularly important for the application of the precautionary approach to fisheries management. There are several methods that can be used to estimate uncertainty (Hilborn and Mangel 1997). Three of the most common methods, likelihood profile, bootstrap, and Bayesian integration, are computationally intense. The likelihood profile and boot strap methods require the objective function to be optimized numerous times, while Bayesian integration requires the model equations to be calculated often millions of times. Because of limited availability of computational resources, these methods are not practical for complex stock assessment models and less computationally intense methods are required. For example, the normal approximation method based on the Hessian matrix is often used (e.g., Fournier et al. 1998).

A common piece of scientific advice considered in fisheries managements is the predicted outcome of future management actions and the uncertainty in these predictions. This requires projecting the population into the future. Unlike estimates of model parameters and historical biomass, estimates of uncertainty for forward projections should also include stochastic variation of demographic processes in the future. However, the main methods used to represent the full uncertainty, both parameter and demographic, the bootstrap and Bayesian integration, are too computationally intense for complex stock assessment models. Less computationally intense methods that are used are often not adequate. For example, methods that project from parameter point estimates ignore parameter uncertainty, which can, in some cases, represent the majority of the uncertainty.

We describe a method for calculating uncertainty in forward projections that includes both parameter and demographic uncertainty that is much less computationally intensive than the bootstrap and Bayesian integration. The method is based on extending the estimation model to include the period of the forward projections and using the normal approximation confidence intervals based on the Hessian matrix. We use simulation analysis to investigate and test this method and compare the results to Bayesian integration and to forward projections from point estimates of the model's parameters. The method is applied to yellowfin tuna in the eastern Pacific Ocean (EPO).

Methods

We describe a method to include uncertainty in forward projections based on the normal approximation using the Hessian matrix by including the projection period as part of the estimation model. This method is implemented in a statistical catch-at-age stock assessment model and tested using simulation analysis. We then apply the method to an age-structured statistical catch-at-length analysis of yellowfin tuna in the eastern Pacific Ocean.

Stock assessment model

A simple catch-at-age stock assessment model is developed. The model is age-structured with two fisheries. Recruitment is related to the stock size using the Beverton-Holt stock-recruitment relationship. The model is fit to total catch and catch-at-age data for each year and fishery. Catch is predicted based on known effort.

The population is assumed to be in an unexploited equilibrium at the start of the modeling time period

(1) $$N_{1,a} = R_0 \exp\left[-M\left(a-1\right)\right]$$

with the last age (A) acting as an accumulating plus group

(2) $$N_{1,A} = \frac{R_0 \exp\left[-M\left(a-1\right)\right]}{1-\exp\left(-M\right)}$$

where $N_{t,a}$ is the number of individuals at time t in age-class a, M is the natural mortality rate, R_0 is the average recruitment in an unexploited population.

Recruitment is assumed to follow the Beverton-Holt stock-recruitment relationship

(3) $$N_{t+1,1} = \frac{S_t}{\alpha + \beta S_t} \exp\left(\varepsilon_{R,t} - 0.5\sigma_R^2\right)$$

(4) $$\alpha = \frac{S_0\left(1-h\right)}{4hR_0}$$

(5) $$\beta = \frac{5h-1}{4hR_0}$$

(6) $$S_t = \sum_a N_a m_a w_a$$

where $\varepsilon_{R,t}$ is the annual recruitment deviate for year t, $-0.5\sigma_R^2$ is the log-normal bias correction factor that makes the expected recruitment equal to the stock-recruitment relationship (otherwise the median recruitment would be equal to the stock recruitment relationship), h is the recruitment as a proportion of R_0 achieved when the spawning biomass (S_t) is 20% of the average spawning biomass in an unexploited population (Francis 1992), w_a is the weight at age a, m_a is the proportion mature at age a.

The abundance is modelled using a simple exponential decay model

(7) $$N_{t+1,a+1} = N_{t,a} \exp\left[-\left(F_{t,a} + M\right)\right]$$

with the last age (A) acting as a accumulating plus group

(8) $$N_{t+1,A} = N_{t,A-1} \exp\left[-\left(F_{t,A-1} + M\right)\right] + N_{t,A} \exp\left[-\left(F_{t,A} + M\right)\right].$$

Fishing mortality is assumed to be separable into age and time specific components and proportional to effort. The fishing mortality is allowed to vary from this relationship using an annual estimated deviate that is penalised using a distributional assumption (Fournier et al. 1998).

(9) $$F_{g,t,a} = q_g E_{g,t} s_{g,a} \exp\left[\varepsilon_{E,g,t} - 0.5\sigma_E^2\right]$$

(10) $$F_{t,a} = \sum_g F_{g,t,a}$$

where $s_{g,a}$ is the selectivity of individuals in age-class a to gear g, $E_{g,t}$ is the effort in time t for gear g, q_g is the catchability coefficient for gear g, and $\varepsilon_{E,g,t}$ is the effort deviate for gear g in year t.

The model is fit to catch data in weight assuming that the observed catch (C^{obs}) is normally distributed around the expected catch (C) with negative log-likelihood (ignoring constants)

(11) $$-\ln L\left(C^{obs} \mid \theta\right) \propto \sum_t \frac{\left[\ln(C_t^{obs}) - \ln(C_t)\right]^2}{2\sigma_C^2}$$

(12) $$C_t = \sum_a C_{t,a} w_a$$

(13) $$C_{t,a} = \frac{F_{t,a}}{F_{t,a} + M_a} N_{t,a}\left[1 - \exp\left(-F_{t,a} - M_a\right)\right]$$

where $C_{g,t,a}$ is the catch in number of individuals from age-class a in time t for gear g, and θ represents the model parameters.

The model is fit to the catch-at-age data using a multinomial-based negative log-likelihood (ignoring constants)

(14) $$-\ln L\left(C_{t,a}^{obs} \mid \theta\right) \propto -\sum_{t,a} C_{t,a}^{obs} \ln\left(p_{t,a}\right)$$

(15) $$p_{t,a} = \frac{C_{t,a}}{\sum_a C_{t,a}}$$

Abundance information for the catch and effort data is modelled using a penalty on the annual effort deviates (ignoring constants and assuming $\sigma_{E,g}$ are known, see Fournier at al. 1998).

(16) $$-\ln P\left(\varepsilon_E\right) \propto \sum_{g,t} \frac{\varepsilon_{E,g,t}^2}{2\sigma_{E,g}^2}$$

Where $\sigma_{E,g}$ is the standard deviation of the logarithm of the annual effort deviates for gear *g*. The size of $\sigma_{E,g}$ determines how much influence the catch and effort data for gear *g* have on the biomass estimates. In general, if $\sigma_{E,g}$ is small the biomass changes to predict the observed catch, and if $\sigma_{E,g}$ is large, the fishing mortality, through $\varepsilon_{E,g,t}$, changes to predict the observed catch.

A penalty is included based on the assumption that recruitment is lognormally distributed around the stock recruitment relationship (ignoring constants)

$$(17) \qquad -\ln P\left(\varepsilon_R\right) \propto n\ln\left(\sigma_R\right) + \sum_t \frac{\varepsilon_{R,t}^2}{2\sigma_R^2}$$

and is equivalent to a prior on the recruitment. Where σ_R is the standard deviation of the logarithm of the annual recruitment deviates and *n* is the number of time periods.

The model estimates the values of R_0, $\varepsilon_{R,t}$, $\varepsilon_{E,g,t}$, σ_R, *h*, and q_g. The values for $s_{g,a}$, *M*, w_a, $\sigma_{E,g}$ and m_a are assumed known (Table 1). To aid the estimation procedure, σ_R was restricted between 0.2 and 2.0.

Normal approximation
The normal approximation method to estimate uncertainty in forward projections is implemented by extending the timeframe of the model for estimation to include the future. As for the past, the recruitment for the future is estimated as an annual deviate with a prior distribution (penalty). The prior distribution is the same as used in the historic period, and describes the uncertainty in future recruitment. However, there are no data in the future to provide information about recruitment. The only additional data included in the model are the effort assumed in the future. Effort deviates for the future are also estimated with the same prior as used in the historical period. The parameters of the stock assessment model are estimated using maximum likelihood estimation (MLE) with the addition of the recruitment deviate and effort deviate penalties. This is equivalent to finding the mode of the joint posterior distribution with locally uniform priors on all other parameters. The confidence intervals for the future abundance are calculated using the normal approximation based on the Hessian matrix (Fournier et al. 1998).

Assuming that the MLE is asymptotically efficient, the Cramer-Rao Lower Bound can be used as an approximation to the true variance of the MLE (Casella and Berger 1990).

$$(16) \qquad \hat{\mathrm{V}}\mathrm{ar}\left(h\left(\hat{\boldsymbol{\theta}}\right)|\boldsymbol{\theta}\right) \approx \frac{\partial h\left(\boldsymbol{\theta}\right)}{\partial \boldsymbol{\theta}}\Big|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} \mathbf{I}^{-1} \frac{\partial h\left(\boldsymbol{\theta}\right)}{\partial \boldsymbol{\theta}}^{\mathrm{T}}\Big|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}$$

where

$$\mathbf{I} = -\frac{\partial}{\partial \boldsymbol{\theta}_i \partial \boldsymbol{\theta}_j} \log L(\boldsymbol{\theta} \mid \mathbf{X})|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}$$

The estimate of the variance can then be used based on asymptotic and regularity conditions to form the confidence interval (Casella and Berger, 1990)

$$(17) \qquad h(\hat{\boldsymbol{\theta}}) - z_{\alpha/2}\sqrt{Var\left(h(\hat{\boldsymbol{\theta}}) \mid \boldsymbol{\theta}\right)} \le h(\boldsymbol{\theta}) \le h(\hat{\boldsymbol{\theta}}) + z_{\alpha/2}\sqrt{Var\left(h(\hat{\boldsymbol{\theta}}) \mid \boldsymbol{\theta}\right)}$$

These confidence intervals with $\alpha = 0.05$ and $z = 1.96$ are used to represent the 95% projection interval of the abundance. We define the projection interval as the interval where there is a 95% probability that the true future value lies within this interval. The frequentist-based confidence interval is used as an approximation of the projection interval.

Bayesian analysis
We use Markov Chain Monte Carlo (MCMC) to estimate the posterior distribution (see Punt and Hilborn 1997). Uniform priors are assumed for all model parameters except for the recruitment and effort deviates, which have normally distributed priors as described above. For each sample of the model parameters from the posterior distribution we project the population forward in time sampling a recruitment deviate each year from the distribution N(0, σ_R) and the effort deviates, $\varepsilon_{E,g,t}$, from the distribution $N \sim (0, \sigma_{E,g})$. The stock recruitment relationship and bias correction factor are used when projecting recruitment. The 5 and 95-percentiles of these projections are used to represent the 95% Bayesian credibility interval of the abundance. More information on Bayesian analysis relevant to this application can be found in Punt and Hilborn (1997) and Maunder and Deriso (2003).

Projections from parameter point estimates
First the parameters of the stock assessment model are estimated using maximum likelihood with the addition of the recruitment deviate and effort deviate penalties. As mentioned above, this is equivalent to finding the mode of the joint posterior distribution. Then, the population is projected forward in time using the MLE parameter values and sampling a recruitment deviate each year from the distribution N(0, σ_R) and effort deviations from $N \sim (0, \sigma_{E,g})$. The stock recruitment relationship and bias correction factors are used when projecting recruitment and effort. The forward projection procedure is repeated numerous times and the 5 and 95-percentiles of these projections are used to represent the 95% projection interval of the abundance. This method is identical to the Bayesian method, but instead of sampling the model parameters from the posterior distribution, the MLE values are used.

Simulation analysis
A simulation model is developed using the same population dynamics model used in the estimation (see Maunder and Deriso 2003 for the general approach). The simulation model is run for 40 years (20 for estimation and 20 for projections) based on known effort (Figure 3) to produce total catch and catch-at-age data using the parameter values in Table 1. Only the first 20 years of catch and catch-at-age data are fit in the estimation procedures. Catchability is modeled

as coming from a log-normal distribution with a standard deviation on the log of the catchability of 0.4 and 0.6 for gears 1 and 2, respectively. The total observed catch is normally distributed around the true catch with a standard deviation of 0.01 and the catch-at-age data are generated using a multinomial distribution with sample size of 50 each year.

Five hundred (two hundred for the Bayesian analysis) artificial data sets are generated and the estimation methods are applied to these data sets. We investigate three methods (Table 2): 1) the normal approximation; 2) Bayesian integration; and 3) forward projections from point estimates. The point estimate method only includes uncertainty in the future recruitment and the effort-fishing mortality relationship. The Bayesian analysis and normal approximation methods also include uncertainty in parameter estimates.

The normal approximation method is further investigated by testing different versions of the recruitment and effort deviate penalties (Table 2): 1) including the full penalty for all years including the future (normal approximation method); 2) dropping $\ln(\sigma_R)$ from the penalty of the future recruitments (estimation adjusted method); and 3) dropping the $\ln(\sigma_R)$ from the penalty of the future recruitments and not using the bias correction factor for the future recruitments (recruitment adjusted method), and 4) the same as (3) except that it excludes the bias correction factor for the effort deviates in the future (fully adjusted method).

We present the percentage of times that the true value lies within, above, and below the 95% projection interval. We also present the % bias when the future abundance is estimated by the MLE estimates from the normal approximation method, the average of the projections for the point estimation method, and the average of the projections for the Bayesian method.

Application

The stock assessments of yellowfin, bigeye, and skipjack tuna in the EPO are some of the most computationally intensive and highly parameterized stock assessment models (Maunder and Watters 2003). The stock assessment model has over one thousand parameters for these applications. Historically, forward projections using this model have been based on parameter point estimates and only include demographic uncertainty for recruitment in the future projections. Therefore, the results ignore parameter uncertainty. Initial analyses using the normal approximation method have been presented for bigeye tuna (Maunder and Harley 2002), but σ_R was fixed, both $\ln(\sigma_R)$ and the bias correction factor where used for the future recruitments, and the normal approximation method had not been tested. We apply the fully adjusted normal approximation method to the stock assessment of yellowfin tuna in the EPO (see Maunder 2002 for a description of the assessment and Maunder and Watters 2003 for further technical details) and compare it to the point estimation method. In both of these models we ignore future variation in catchability represented by effort deviates. This is because fishing mortality is not proportional to effort for some of the fisheries and therefore the standard deviation for the effort deviates in these fisheries is fixed at a relatively large value to remove any biomass information from the catch and effort data for these fisheries. Not including effort deviates in the future will under estimate the uncertainty. This model has 1919 parameters and takes 7 hours to converge on a 2.8 Ghz Pentium 4 desktop computer.

Results

Simulations

The coverage of the biomass projection interval for the point estimation method is very poor in the first few years with a large number of true biomass points being outside the projection interval (Table 3). The coverage becomes close to the desired coverage as the projection time frame is increased, but is still slightly lower than the desired level. The normal approximation method has better coverage for the first few years, but has poorer coverage as the projection time frame is increased. The projection intervals are asymmetrical with a greater chance of the true value being above the projection interval. The coverage is changed, but not improved by removing $\ln(\sigma_R)$ from the penalty on the recruitment deviation in the projection time period in the estimation adjusted method. The coverage is greatly improved and close to the desired level if both $\ln(\sigma_R)$ is removed from the penalty on the recruitment deviations and the bias correction factor is removed in the projection time period in the recruitment adjusted method. However, the projection intervals remain asymmetrical in there coverage. The fully adjusted method that also removes the effort deviate bias correction factor in the future, performs about the same as the recruitment adjusted method. The coverage of the Bayesian method is higher than the desired level and higher than the other methods. However, the coverage performance is symmetrical, with equal numbers of true biomasses above and below the projection intervals.

The size of the projection intervals are smaller for the normal approximation method compared to the point based method, except for the first few years. However, the projection intervals are similar in size for the fully adjusted normal approximation method compared to the point based method, except the first few years. The confidence intervals are largest for the Bayesian method. Figure 4 shows the projection distribution for the different methods for a single simulated data set and highlights the difference in projection uncertainty and the asymmetrical nature of the Bayesian and point estimation methods.

There is significant bias in all four of the normal approximation methods' estimates of future spawning biomass (Figure 5). The bias correction factor and the $\ln(\sigma_R)$ for the future recruitment, and the bias correction factor for the effort deviates cause the spawning biomass to be negatively biased. However, when these are removed, there is positive bias in the spawning biomass. This bias can be attributed to the stock recruitment relationship (Figure 6). Bias in estimated recruitment follows a similar pattern to bias in spawning biomass (Figure 7), and recruitment does not appear to be biased when the steepness is set to 1 (recruitment is independent of stock size) for the fully adjusted normal approximation method (Figure 8).

The estimates of σ_R are negatively biased for all the methods except for the Bayesian method (Figure 9). The negative bias is much greater and R_0 is also negatively biased (Figure 10) if $\ln(\sigma_R)$ is included in the recruitment deviate penalty in the projection period.

Application

The spawning biomass confidence intervals of the fully adjusted normal approximation method were much larger in the future projections than for the historic period (Figure 11). The mean of the point estimate method projections was essentially identical to the MLE of the fully adjusted normal approximation method. The confidence interval from the point estimate method took about two years to become as wide as the fully adjusted normal approximation method. The lower bound of the confidence interval estimated by the point estimate method was not as low as the fully adjusted normal approximation method.

Discussion

There are several biases that are introduced by the normal approximation method: 1) bias in the estimation of σ_R if $\ln(\sigma_R)$ is included in the penalty for the future recruitment deviates, 2) bias in the projections due to the inclusion of the bias correction factor in the future for either recruitment or effort deviates, and 3) bias in projected recruitment due to the stock recruitment relationship.

1) The estimation routine minimizes the negative log likelihood which includes the term ln(σ_R). However, for the future recruitment there is no additional information to provide estimates of the recruitment deviation. Therefore, the model parameter estimates should be the same as those achieved when the model is fit without the projection period. This is not the case because minimizing the negative log likelihood with ln(σ_R) for these years causes the σ_R to be estimated lower. This is because there are additional years where $\varepsilon_{R,t}^2 / 2\sigma_R^2 = 0$ due to no information in the data about $\varepsilon_{R,t}$, so that a smaller value for σ_R reduces the penalty by reducing $\ln(\sigma_R)$. This result highlights a consequence of the approach commonly used in fisheries to estimate the recruitment residuals as free parameters. If the model includes years where there is no or little information, the estimate of σ_R would be biased low. This is consistent with the results of Maunder and Deriso (2003) and methods that integrate out the recruitment deviations may be better estimators of σ_R (see Maunder and Deriso 2003).

2) The bias correction factor is added to keep the expected recruitment equal to the stock recruitment relationship. This assumes that all the recruitments are iid lognormal. However, if there is little information in the data about recruitment for certain years for recruitment, these years will be estimated at values below the average due to the effect of the bias correction factor and that the penalty on the recruitment deviates is centered at zero. Therefore, because there is no information in the future, recruitment in the future will be biased low if the bias correction factor is used. This result highlights an additional consequence of the approach commonly used in fisheries to estimate the recruitment residuals as free parameters. If the model includes years where there is no or little information, the estimate of recruitment will be biased low for these years.

3) Deterministic projections of a population dynamics model will underestimate the effect of a stock recruitment relationship. In a stochastic projection when random recruitment is lower than that expected from the stock recruitment relationship (i.e. a negative annual recruitment deviate), the spawning biomass will also reduce more than expected causing the stock-recruitment relationship to make the expected recruitment lower in following years. The opposite also occurs when the recruitment deviate is positive. However, due to the shape of the Beverton-Holt stock recruitment curve, this has more impact for lower than expected recruitment. Without stochastic recruitment this does not occur and therefore the normal approximation method will, if correct

for other biases, on average over estimate recruitment in the future if there is a Beverton-Holt stock recruitment relationship.

Maunder and Deriso (2003) found that estimation of σ_R was possible when maximizing the penalized likelihood, but estimability was reduced when the catch-at-age data was not available for all years. They showed that a local optimum often occurs close to the true value of σ_R, but a global optimum occurs at zero. Therefore, putting reasonable bounds on σ_R and initiating the estimation routine at a reasonable value for σ_R, as in this analysis, may provide reasonable results. If the estimate of σ_R is at the bound, estimation methods that integrate over the annual recruitment deviates (e.g., Bayesian integration) may be needed (see Maunder and Deriso 2003). In collaboration with Dave Fournier (Otter Research) we have shown that the Laplace approximation (as implemented in ADMB) performs identically to the numerical integration method of Maunder and Deriso (2003), but is an order of magnitude faster and may be a promising method to estimate σ_R in these cases. The Laplace approximation method may also improve the normal approximation method for projections (note that the adjustments presented in this study may not be appropriate if using the Laplace approximation).

The yellowfin tuna application highlighted a problem with the normal approximation method due to the method used to include abundance information from catch and effort data. As explained previously, the amount of information about abundance contained in the catch and effort data is controlled by the standard deviation of the penalty on the effort deviates. However, this standard deviation is also used to determine the variability in catchability in the future. So down weighting the information about abundance contained in the catch and effort data by increasing the standard deviation will increase the variability of catchability in the future. The main reasons to down weight the information about abundance contained in the catch and effort data is because catchability changes over time. To overcome this, catchability could be modeled as a random walk process (e.g. Hampton and Fournier 2001; Maunder and Watters 2003) or the assumption that fishing mortality is proportional to effort could be modified.

The normal approximation requires the confidence intervals to be symmetrical. In many cases symmetry may not be appropriate. For example, biomass cannot be less than zero and this may contribute to the asymmetry coverage of the normal approximation methods in this analysis. However, calculating the confidence intervals on the natural logarithm of biomass and then using the appropriate transformation, as done by Hampton and Fournier (2001), may improve coverage. The approach of including the projection period in the estimation model can be used in a likelihood profile context to produce asymmetrical confidence intervals. However, this requires optimizing the objective function numerous times, which may be impractical due to computational or time limitations.

We used the normal approximation method to reduce the computational demands of performing forward projections while still allowing for both parameter uncertainty and future demographic stochasticity. Alternative methods to reduce computational demands are possible. For example, the parameter estimates could be sampled from a multivariate normal distribution based on the Hessian matrix and these used to do stochastic forward projections (Patterson et al. 2001). Methods to reduce computational demands that can be used with Bayesian integration or bootstrap procedures are also available. The stock assessment method used for our analyses follows the method of Fournier et al. (1998) and allows for continuous fishing and natural mortality throughout the year using the Baranov catch equation and estimating annual effort deviates. An alternative method to implement the Baranov catch equation is to iteratively solve the catch equation within each function evaluation of the parameter estimation procedure. However, this method is also computationally demanding. The computational demands can be reduced by approximating continuous fishing and natural mortality by removing catch in the middle of the year (the Pope approximation). This removes the need to estimate the annual effort deviates, but also changes the method used to include the information from the catch and effort data. Maunder and Starr (2001) used this method to implement a Bayesian analysis for a catch-at-age model with several gears. The reduced computational demands come at the price of possible bias due to the approximation. However, it should be noted that fishing and natural mortality do not occur at a constant rate throughout the year and therefore the method of Fournier et al. (1998) is also only an approximation. It also should be noted that the normal approximation method for forward projections does not work when using the Pope approximation (taking catch out in the middle of the year and not modeling the effort-fishing mortality relationship) because the future biomass may become negative which causes computational problems. Corrections for this are possible (e.g. penalizing the biomass to be positive), but may cause biased parameter estimates (e.g. overestimate average recruitment). Similarly, problems occur when implement harvest strategies based on total catch using the normal approximation method for forward projection because this requires fitting to the future catch when using the of Fournier et al. (1998) method and may produce different parameter estimates.

We have presented a method for including both parameter uncertainty and demographic stochasticity in forward projections that is less computationally intensive than standard techniques (Bayesian and bootstrap). This method can be used for the highly computationally intensive models used for many modern stock assessments. The methods includes the projection period in the estimation model, estimate future recruitment using penalized recruitment deviates, and use normal approximation based on the Hessian matrix to estimate projection intervals. To reduce bias the $\ln(\sigma_R)$ term in the recruitment residual penalty should be removed from the projections if σ_R is estimated and the bias correction term should not be used for future recruitment or effort deviates. If the model includes a Beverton-Holt stock-recruitment relationship, the results will be biased.

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Table 1. Parameter values used in the simulator and the estimator. If a value for a parameter is assumed for the estimator it is set equal to the value for the simulator.

Parameter	Simulator value	Estimated
R_0	1000	Estimated
ε_R	$N(0,\sigma_R)$	Estimated
$\varepsilon_{E,1}$	$N(0,\sigma_{E,1})$	Estimated
$\varepsilon_{E,2}$	$N(0,\sigma_{E,2})$	Estimated
σ_R	1.0	Estimated
h	0.6	Estimated
q_1		Estimated
q_2		Estimated
$\sigma_{E,1}$	0.4	Assumed
$\sigma_{E,2}$	0.6	Assumed
s_1, s_2	see Figure 1	Assumed
M	0.3	Assumed
w	see Figure 2	Assumed
m	0 below x and 1 above	Assumed

Table 2. Description of the various models that were used in the simulation study.

	Method	Rdev bias correction	Edev bias correction	Rdev likelihood
Model1 - Point	Point-based	Yes	Yes	NA
Model2 - Normal	Normal approx.	Yes	Yes	Yes
Model3 - Estimation adjusted	Normal approx.	Yes	Yes	No
Model4 - Recruitment adjusted	Normal approx.	No	Yes	No
Model5 - Recruitment and effort adjusted	Normal approx.	No	No	No
Model6 - Bayesian	Bayesian	Yes	Yes	NA

Table 3. Estimates of probability interval coverage for the spawning biomass for projections of 2, 5, and 10 year using the three different methods and the three alternative normal approximation methods a) only bias correction in future, b) include neither log(sigmaR) or bias correction, and c) similar to b, but with no bias correction on future effort deviates. See Table 2 for descriptions of the models.

	model 1		
	2 years	5 years	10 years
below	0.16	0.05	0.04
inside	0.53	0.94	0.94
above	0.31	0.02	0.03
size	48	287	298
	model 2		
below	0.02	0.02	0.02
inside	0.92	0.84	0.80
above	0.06	0.14	0.18
size	131	177	173
	model 3		
below	0.01	0.00	0.00
inside	0.92	0.82	0.78
above	0.07	0.18	0.22
size	131	188	181
	model 4		
below	0.01	0.00	0.00
inside	0.92	0.96	0.96
above	0.07	0.04	0.04
size	131	274	297
	model 5		
below	0	0	0
inside	0.92	0.95	0.94
above	0.08	0.05	0.06
size	129	263	278
	model 6		
below	0.02	0.02	0.02
inside	0.96	0.97	0.97
above	0.02	0.02	0.02
size	135	343	348

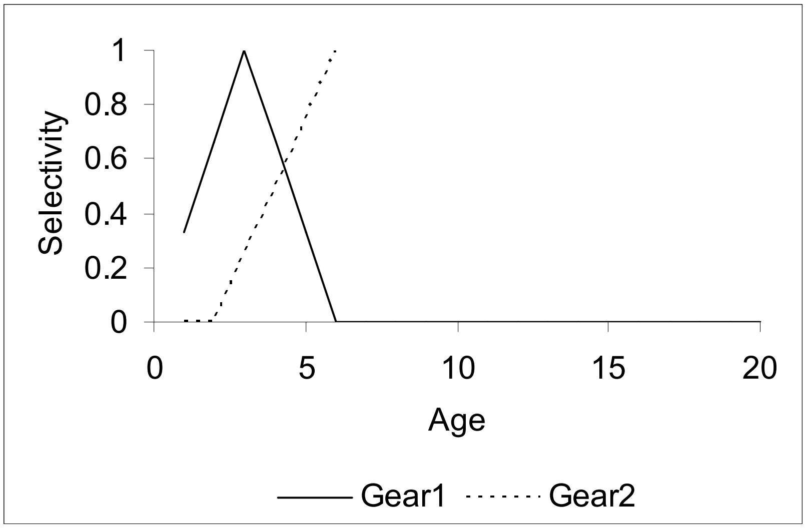

Figure 1. Selectivity for the two fisheries used in the simulation analysis.

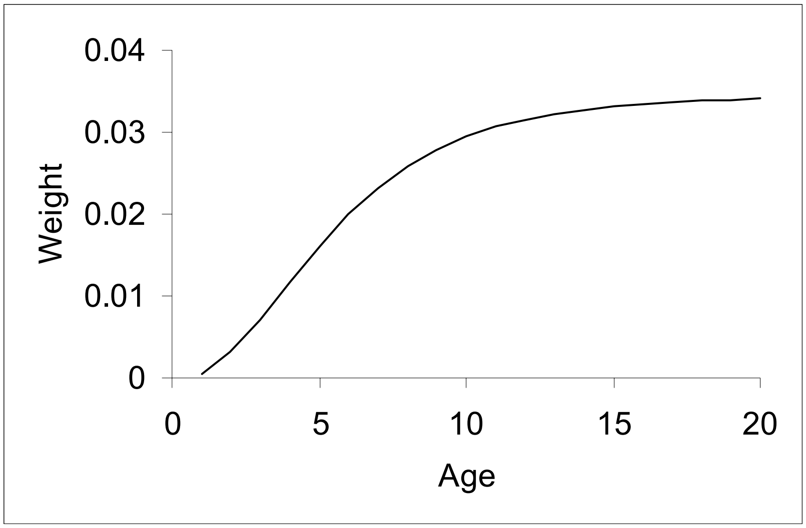

Figure 2. Weight at age used in the simulation analysis.

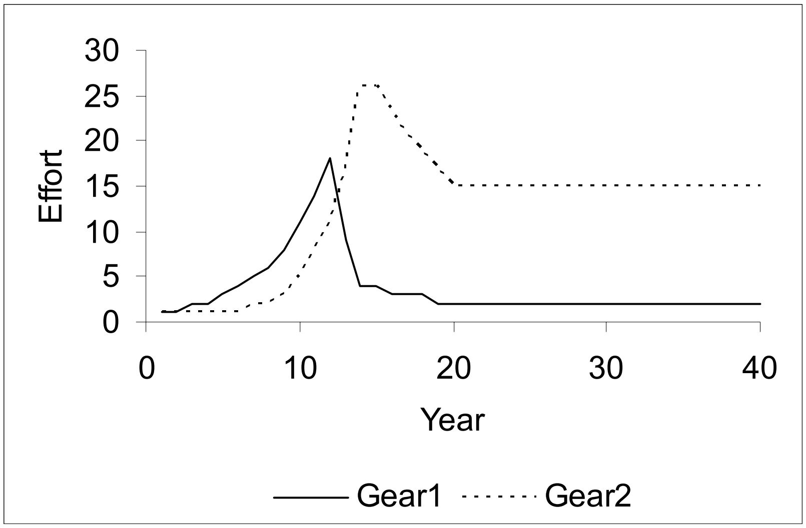

Figure 3. Effort values used for the two fisheries in the simulation analysis.

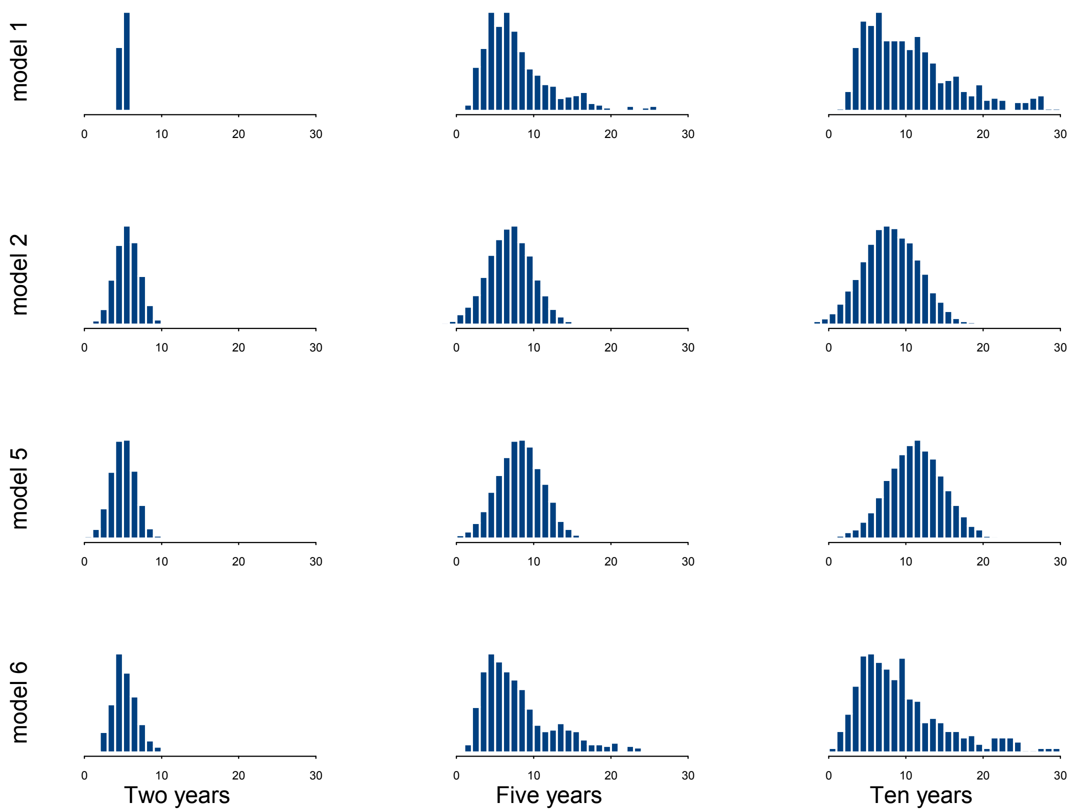

Figure 4. Comparisons of the projection distribution of spawning stock biomass from the different estimation methods for a single simulated data set. The true values in this simulation were 4.3, 5.2, and 9.3, for two, five, and ten years into the future.

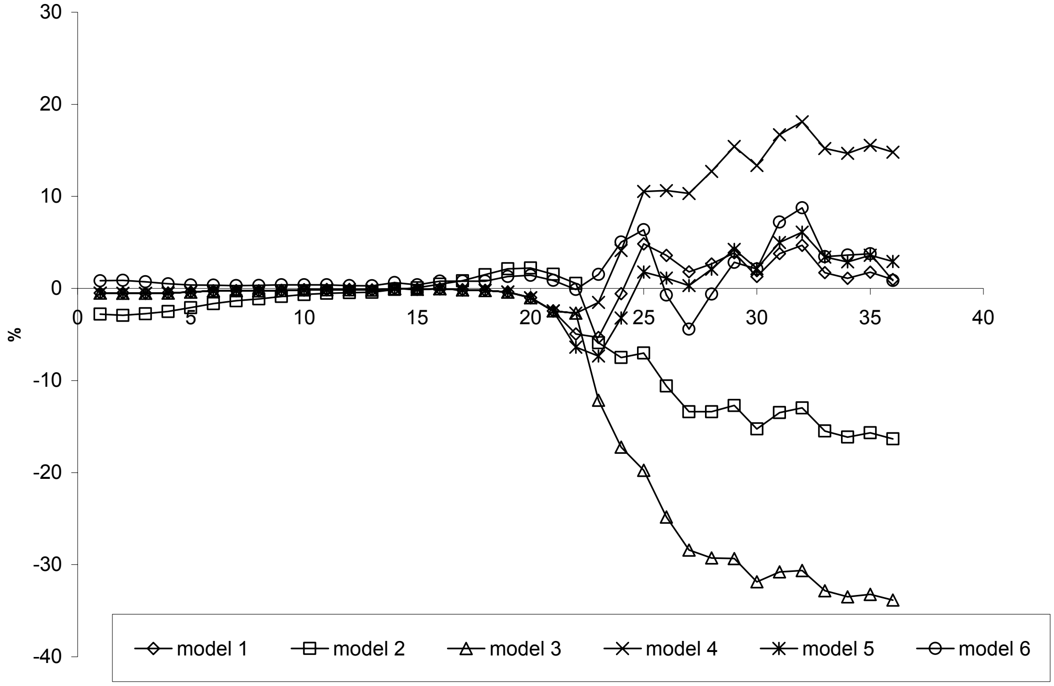

Figure 5. Bias in the estimates of spawning stock biomass.

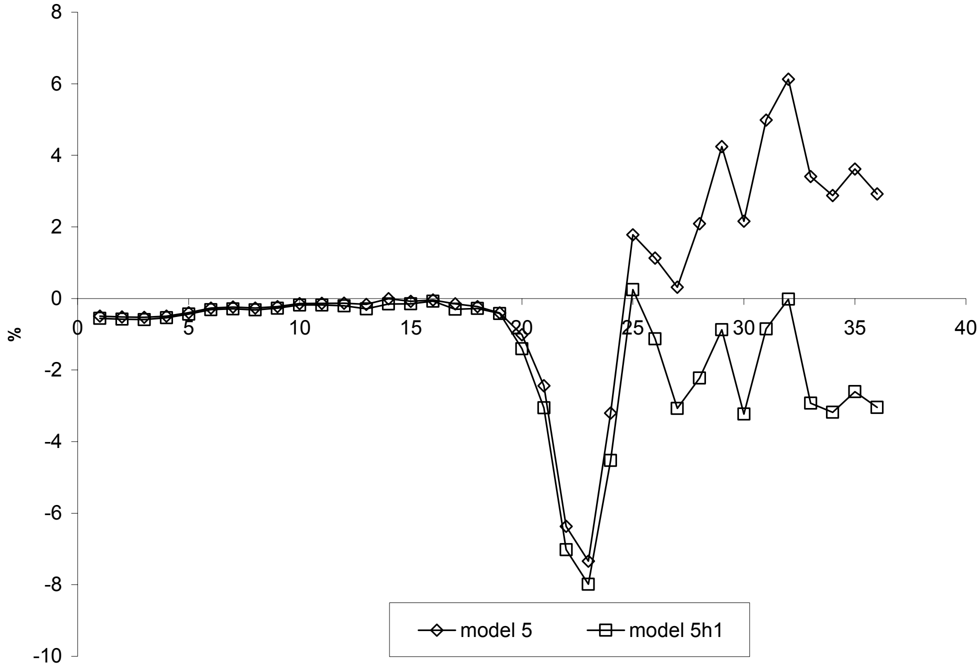

Figure 6. Bias in the estimates of spawning stock biomass using the fully adjusted normal approximation method when the model has a Beverton-Holt stock recruitment relationship and when recruitment is independent of stock size.

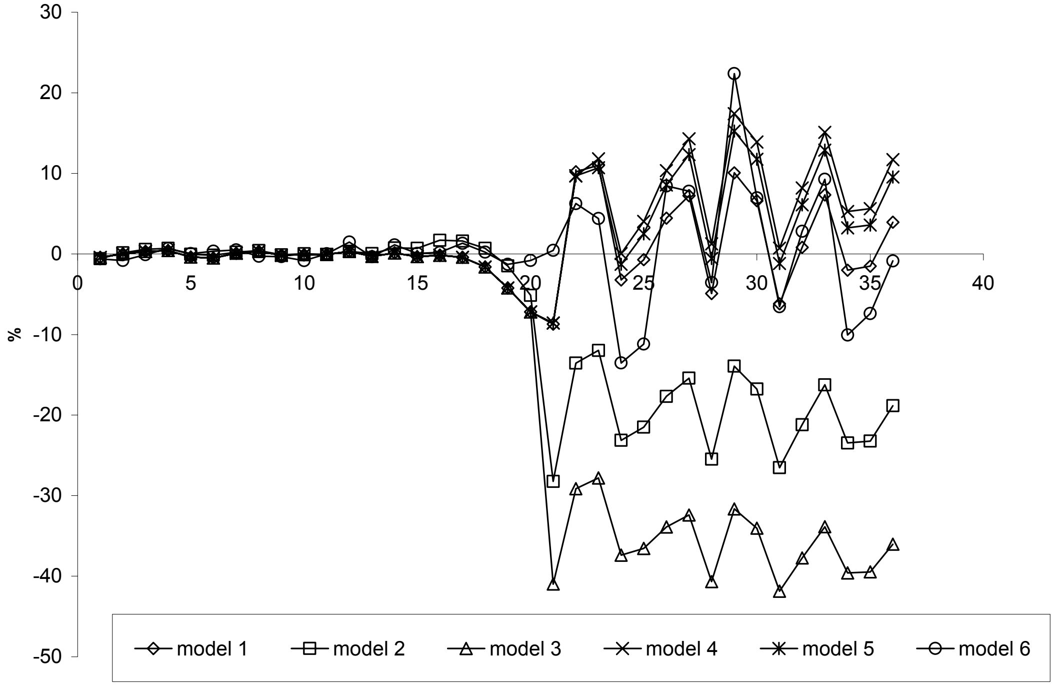

Figure 7. Bias in the estimates of recruitment.

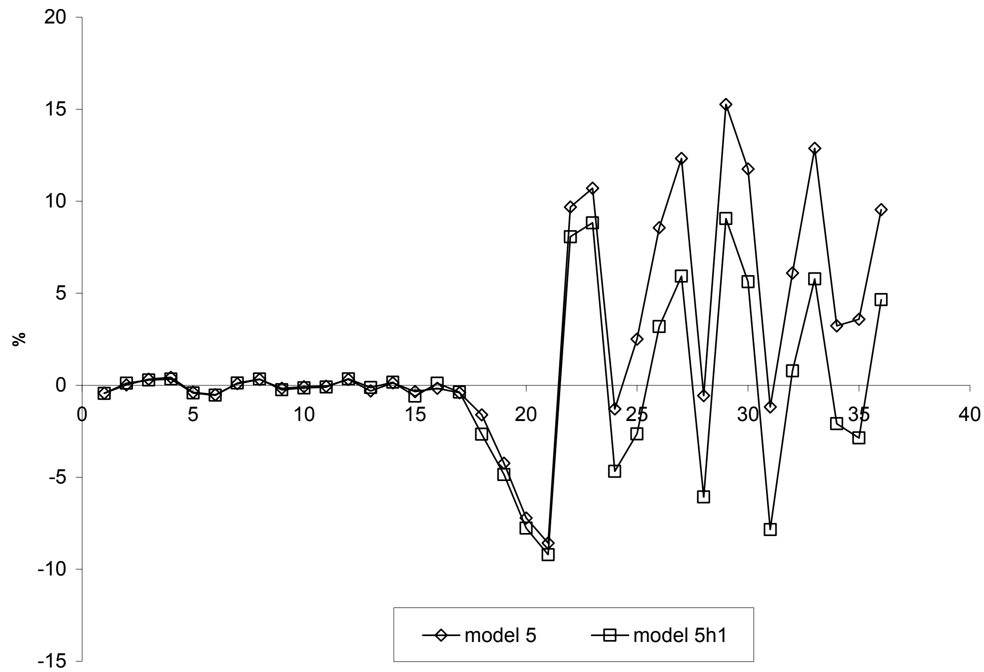

Figure 8. Bias in the estimates of spawning stock biomass using the fully adjusted normal approximation method when the model has a Beverton-Holt stock recruitment relationship and when recruitment is independent of stock size.

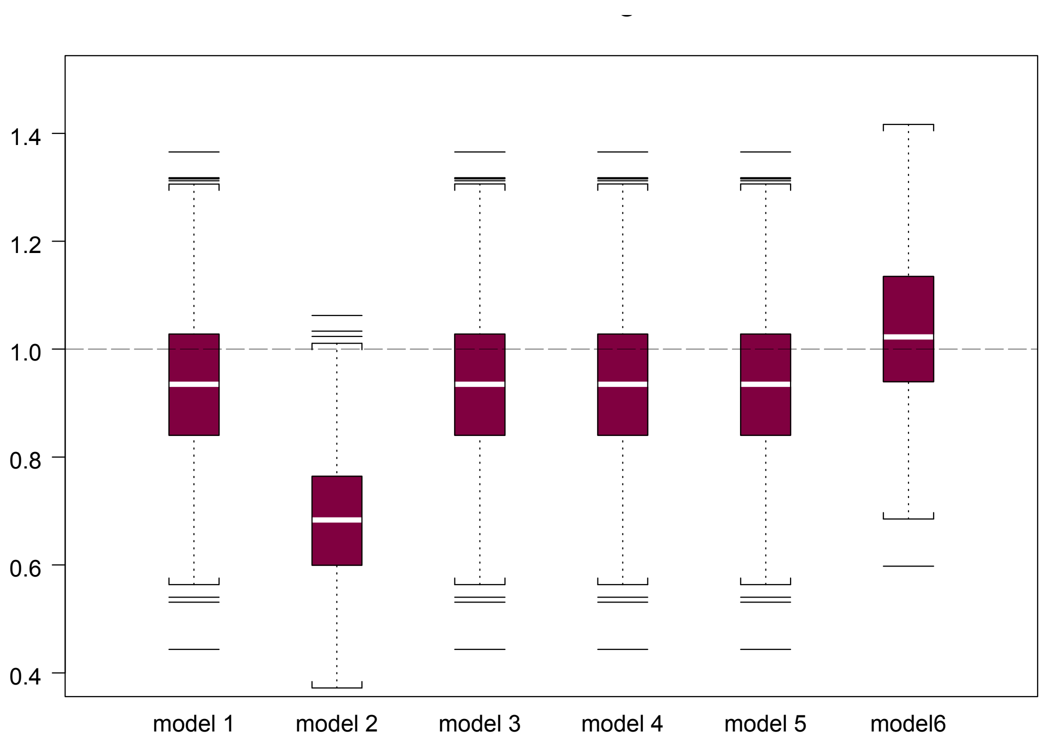

Figure 9. Estimates of σ_R for each simulated data set using the different estimation methods.

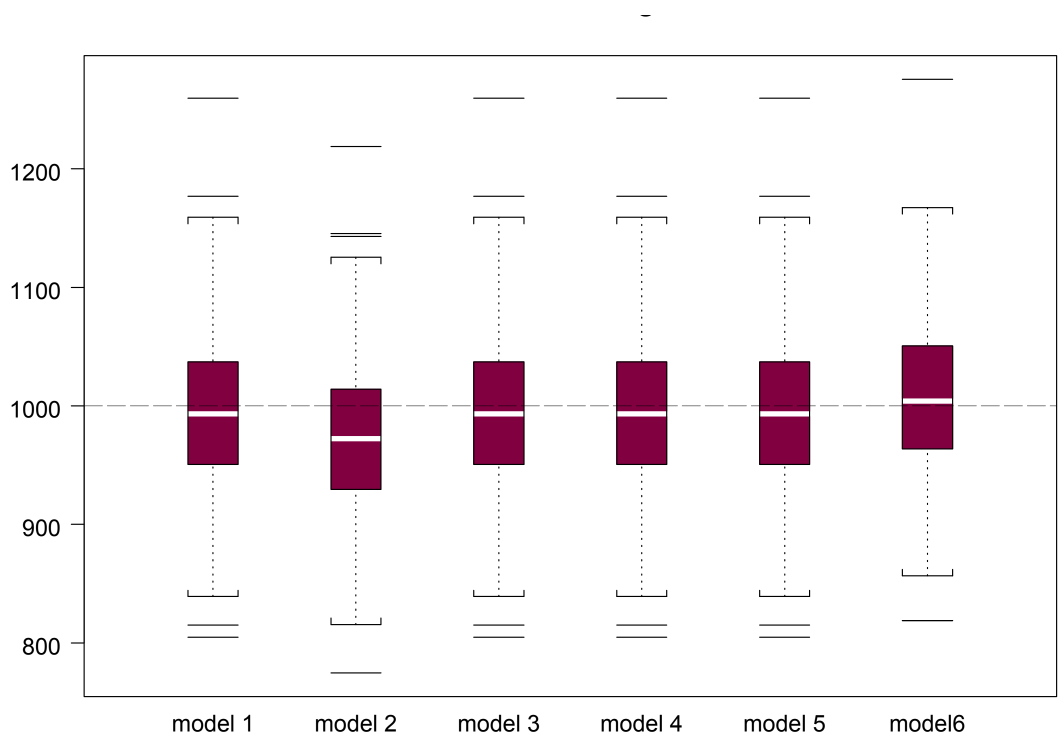

Figure 10. Estimates of R_0 for each simulated data set using the different estimation methods.

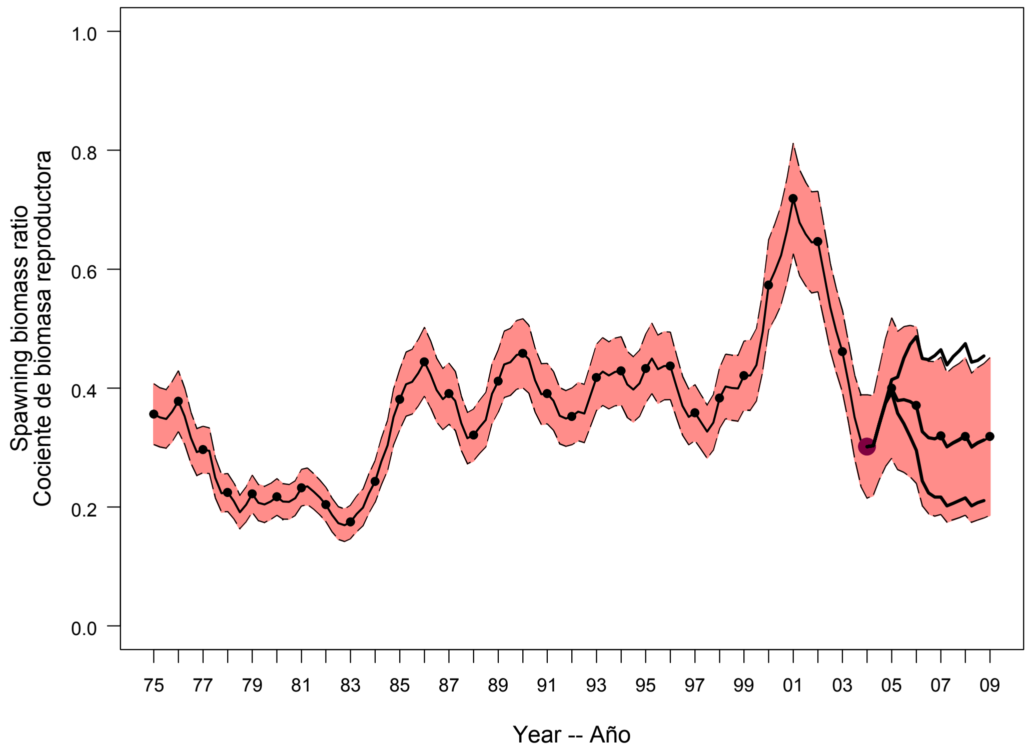

Figure 11. Comparisons of estimates and projection intervals for spawning biomass ratio from the point estimate and fully adjusted estimation methods for yellowfin tuna in the eastern Pacific Ocean. The shaded area represents the 95% confidence intervals for the fully adjusted estimation method, the thin line with solid points represents the MLE estimates from the fully adjusted estimation method, the large solid circle represents the end of the historical estimation period, and the solid lines represent the 2.5% and 97.5% percentiles and the mean from the point estimation method.